THERMAL STATE OF ORGANIC LASER-IRRADIATED TISSUE

V. N. Vasiliev and S. K. Serkov UDC 536.3

Consideration is given to laser interaction between UV laser radiation and organic tissue. This problem is solved in two steps: determination of radiation intensity fields at scattering and partial absorption in the tissue and temperature distribution determination. A nonstationary energy equation is solved by analytical methods to determine the thermal field in the tissue to the temperature of phase transformation of water. The analytical solution is compared with the numerical one obtained by the method of finite elements.

Interaction between laser radiation and a substance is the subject matter in many works [1, 2]. Design and application of new laser scalpels in medicine poses new problems. A laser scalpel represents a system actuating a pulse laser, a radiation transmission line (a single fiber in the simplest case) and a focusing system which allows different power density distribution to be obtained in the tissue under irradiation. As a pulse laser, use may be made, for instance, of a neodymium laser operating at a wavelength of 1.06 μm (E = 200 mJ, τ_p = 200 μsec, ν = 20-200 Hz) or a holmium laser with a wavelength of 2.8 μm. The necessity of displacement from the near to far UV-radiation region is connected with increasing the absorption coefficient and, consequently, a higher efficiency of the tool.

The basic characteristics the effectiveness of using a laser are the rates of tissue heating and its destruction. In connection with this, the problem of calculating the nonstationary temperature field of the tissue exposed to radiation of the laser scalpel arises.

The general statement of the problem on the laser scalpel–tissue involves a study of many processes:

phase transitions occurring during tissue destruction (protein denaturation, water evaporation, carbon skeleton failure);

mass transfer processes (transpiration of gases through the skeleton, carrying out the particles of the failed tissue);

radiation attenuation in the medium (absorption and scattering coefficients are functions of temperature);

thermal superradiation from the heated tip of the laser scalpel;

tissue burning.

The present article is concerned with the problem of tissue heating to the boiling point of water and analysis of the influence of heat transfer by conduction, convection, and radiation on the main process. The problem requires separate treatment because just the zone of temperatures below 100°C determines the zone of tissue necrosis and blood coagulability.

1. Propagation of laser radiation in tissue. Organic tissue is an absorptive-scattering medium, the radiation field in which is described by the following integrodifferential equation [1]

$$\frac{dI}{ds} + [\varkappa(s) + \sigma(s)] I = \frac{1}{4\pi} \sigma(s) \int\limits_{\bar{\Omega}'=4\pi} \psi(\bar{\Omega}\bar{\Omega}') I d\bar{\Omega}'. \quad (1)$$

This equation may be solved by finite difference or statistical methods. However this is another problem and in virtue of nonlinearity of the equation and uncertainty of coefficients its solution has not been obtained yet. Analogous problems are discussed in [2, 3] where it is shown that the scattering process in the tissue may be conventionally presented as occurring in two zones (Fig. 1): intermediate zone I + II, in which the surface radiation source is transformed into a spherically symmetrical one and spherical scattering zone III with radiation being uniform in all directions. Figure 1 illustrates radiation propagation in the tissue. Radiation is emitted from a laser scalpel tip with a divergence angle of 15-20° and forms a light spot (a surface source) with a radius a on an irradiated tissue. Next, penetrating into the tissue, the radiation is scattered and partially absorbed (zones I $\{(r, z, \varphi): z^2 + r^2 < a^2, z > 0\}$ and II $\{(r, z, \varphi): z^2 + r^2 > a^2, r < z \tan\beta, r^2 + (z - l)^2 < (a + l \tan\beta)^2\ z > 0\}$. As a result, a spherically symmetrical radiation source "Sphere" is formed $((0, l), a + l \tan\beta)$. Zone III $\{(r, z, \varphi): r^2 +$

Institute of Precise Mechanics and Optics, St. Petersburg, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 63, No. 4, pp. 449-456, October, 1992. Original article submitted March 19, 1991.

 1062-0125/92/6304-1016$12.50

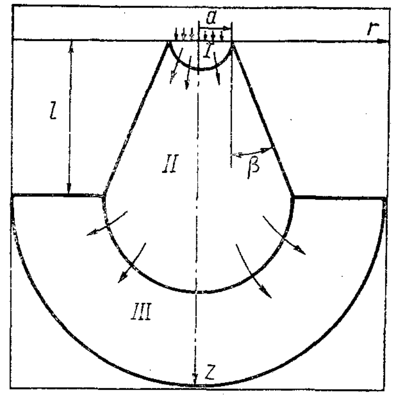

Fig. 1. Schematic of radiation propagation in an absorbing-scattering tissue.

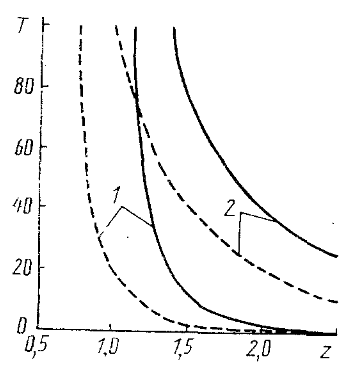

Fig. 2

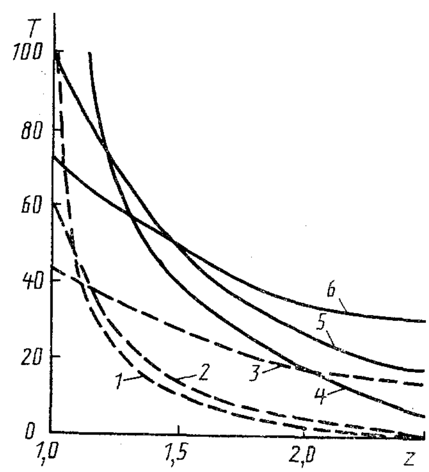

Fig. 3

Fig. 2. Temperature distribution T, °C in organic tissue calculated by formulas (12) (dashed curves) and (16) (solid curves): 1) $\tau = 0.5$ sec; 2) 1 sec. z, mm.

Fig. 3. Temperature distribution obtained by the method of finite elements: 1) $\tau = 0.5$ sec; $\gamma = 1$; 2) 0.5 and 0.5; 3) 0.5 and 0; 4) 1 and 1; 5) 1 and 0.5; 6) 1 and 0.

$(z - l)^2 > (a + l \tan \beta)^2$, $z > l\}$ is the zone of spherical scattering, light intensity is uniform in all directions, radiation undergoes only the Bouguer attenuation. In the tissue attenuating the radiation there is no zone III, $\beta = 0$, $l \to \infty$. A strongly scattering tissue has no zone II, $l = 0$.

To determine the radiation intensity quantitatively, we assume that the constant radiation intensity lines (isophones) are of a spherical form. Set up an equation of energy balance for a volume element enclosed by two nearby isophones. A difference of fluxes passing through the isophones is the power absorbed in the substance volume according to the Bouguer law:

$$\int_S p(\xi)\, dS - \int_{S'} p(\xi')\, dS = \int_{dV} p(\xi)(\varkappa + \sigma)\, dV. \tag{2}$$

Reducing this equation to a differential one (at $\kappa \gg \sigma$):

$$\frac{d\,[p(\xi)\, S]}{dV} = p(\xi)\, \varkappa,$$

we find the radiation power density function $p = p(\xi)$, where ξ is the independent variable, is the same at any point of an isophone and unambiguously related with coordinates.

Assume the power density in the light spot on an irradiated tissue to be uniform and equal to $E\nu/(\pi a^2)$. Then for the power density fields inside the tissue, we may obtain the following expressions:

zone I:

$$p(h) = \frac{E\nu}{\pi(h^2 + a^2)} \exp\left(-\frac{\varkappa h}{2}\right), \quad h = \overline{0, a},$$
$$h = \frac{z^2 + r^2 - a^2 + \sqrt{(z^2 + r^2 - a^2)^2 + 4a^2z^2}}{2z}; \tag{3}$$

zone II:

$$p(t) = \frac{E\nu \exp\left[-\varkappa\left(t + \frac{a}{2}\right)\right]}{2\pi(a + t\,\mathrm{tg}\,\beta)^2}, \quad t = \overline{0, l}, \tag{4}$$
$$t = \frac{z + a\,\mathrm{tg}\,\beta - \sqrt{(z + a\,\mathrm{tg}\,\beta)^2 - (1 - \mathrm{tg}^2\beta)(z^2 + r^2 - a^2)}}{(1 - \mathrm{tg}^2\beta)};$$

zone III:

$$p(R) = \frac{E\nu \exp\left\{-\varkappa\left[l(1 - \mathrm{tg}\,\beta)\right] + \frac{a}{2} + \sqrt{(z - l)^2 + r^2}\right\}}{2\pi[(z - l)^2 + r^2]}, \tag{5}$$
$$R = \sqrt{(z - l)^2 + r^2}, \quad R > a + l\,\mathrm{tg}\,\beta.$$

Note that for a strongly absorbing tissue ($\kappa > 100\ \mathrm{cm}^{-1}$), we may consider that zones II and III do not exist. In this case, all the radiation is concentrated in near-surface zone I:

$$p(r, z) = \begin{cases} \frac{E\nu}{\pi a^2} \exp(-\varkappa z), & r \leqslant a; \\ 0, & r > a. \end{cases} \tag{6}$$

This is valid for a holmium laser emitting in the far UV-radiation zone.

2. Temperature field of tissue. Determination of temporal and spatial temperature field distribution is reduced to solution of the nonstationary energy transfer equation with allowance for radiation [2]

$$c\rho \frac{\partial T}{\partial \tau} = \lambda \Delta T - \mathrm{div} \int \bar{\Omega} I d\bar{\Omega}. \tag{7}$$

The addend in the r.h.s. takes into account laser radiation absorption and considering (2) it may be written as

$$w = \frac{\int_{dV} \varkappa p(r, z)\, dV}{dV} = \varkappa p(r, z). \tag{8}$$

Consideration will be given to tissue heating only to 100°C. In the temperature range 20-100°C, coefficients c, ρ, λ, and κ may be treated as constant quantities. Organic tissue represents a complex composite material, its properties may be determined experimentally or taken close to the properties of water ($\lambda = 0.597$ W(m·K), $c_p = 4.18 \cdot 10^6$ J/(m^3·K), $\kappa = 150\ \mathrm{m}^{-1}$, $\tilde{\lambda} \rightarrow = 1.06\ \mu$m).

At first, consider a temperature field of a strongly absorbing medium. Then the density of internal sources is $w = (E \cdot \nu)/(\pi a^2)$, which corresponds to the surface source $E\nu/(\pi a^2)$.

As a surface source, we may take a point source continuously acting in time with an intensity $E\nu$ (time-averaged pulses). A set of equations which describe such a process is

$$c\rho \frac{\partial T}{\partial \tau} = \lambda \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + E\nu\delta(r, z), \tag{9}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r\to\infty} = \left.\frac{\partial T}{\partial z}\right|_{z\to\infty} = 0, \tag{10}$$

$$\lambda \frac{\partial T}{\partial z} - \alpha T|_{z=0} = 0. \tag{11}$$

Without heat transfer ($\alpha = 0$, $\mathrm{Nu} \ll 1$), a solution of the problem (9)-(11) is known [4] as

$$T_0 = \frac{E\nu}{4\pi\lambda \sqrt{r^2 + z^2}} \operatorname{erfc} \frac{\sqrt{c\rho (r^2 + z^2)}}{\sqrt{4\lambda\tau}}. \tag{12}$$

Find an analytical solution of the problem (9)-(11) at $\alpha \neq 0$. Write the solution of the problem in the form $T = T_0 + T_1$, then

$$\lambda \frac{\partial T_1}{\partial z} - \alpha T_1|_{z=0} = -\lambda \frac{\partial T_0}{\partial z} + \alpha T_0|_{z=0} = \frac{\alpha E\nu}{4\pi\lambda R} \operatorname{erfc}\left(\frac{R\sqrt{c\rho}}{\sqrt{4\lambda\tau}} \right). \tag{11'}$$

Here $R = \sqrt{r^2 + z^2}$.

Applying the Laplace and Hankel transformation [5] to the system (9)-(11), we arrive at a linear equation for an unknown z:

$$\begin{cases} s_\tau \tilde{T}_1 = \dfrac{\lambda}{c\rho} \dfrac{d^2 \tilde{T}_1}{dz^2} + \dfrac{\lambda}{c\rho} s_r^2 \tilde{T}_1, \\ \lambda \dfrac{\partial \tilde{T}_1}{\partial z} - \alpha \tilde{T}_1 = \dfrac{\alpha E\nu}{4\pi\lambda s_\tau} \left(s_r^2 + \dfrac{c\rho s_\tau}{\lambda} \right)^{-1/2}, \end{cases}$$

$$\tilde{T}_1 = \tilde{T}_1(z, s_\tau, s_r).$$

The solution of the system is

$$\tilde{T}_1 = \frac{\alpha E\nu}{4\pi\lambda s_\tau} \left(s_r^2 + \frac{c\rho s_\tau}{\lambda} \right)^{-1/2} \left(-\lambda \sqrt{s_r^2 + \frac{c\rho s_\tau}{\lambda}} - \alpha \right)^{-1} \times \exp\left(-\sqrt{s_r^2 + \frac{c\rho s_\tau}{\lambda}}\, z \right).$$

Passing from a transform to an inverse transform, we obtain

$$T_1 = -\frac{\alpha E\nu}{4\pi c\rho\lambda} \exp\left(\frac{\alpha z}{\lambda} \right) \int_0^\tau \exp\left(\frac{\alpha^2 \tau}{c\rho\lambda} \right) \operatorname{erfc}\left(\frac{\alpha\sqrt{\tau}}{\sqrt{c\rho\lambda}} + \frac{z\sqrt{c\rho}}{2\sqrt{\lambda\tau}} \right) \times$$

$$\times \int_0^\infty s_r \exp\left(-\frac{\lambda s_r^2 \tau}{c\rho} \right) J_0(r s_r)\, ds_r\, d\tau.$$

Using asymptotic estimates of the integrand and the mean-value theorem to evaluate integrals, we may write

$$\Lambda = \exp\left(\frac{\alpha^2\tau^*}{c\rho\lambda}\right)\operatorname{erfc}\left(\frac{z\sqrt{c\rho}}{2\sqrt{\lambda\tau^*}}\right) s_r^* \exp\left(-\frac{\lambda s_r^{*2}\tau^*}{c\rho}\right) J_0(rs_r^*)\,\tau s_r.$$

In virtue of the rapidly decreasing exponential function, $s_r = \sqrt{c\rho}/\sqrt{\lambda\tau}$, and τ^* is chosen so that Λ is maximal. At $\tau < (\lambda c\rho/a^2)$

$\Lambda \leqslant \operatorname{erfc}\left(\frac{z\sqrt{c\rho}}{2\sqrt{\lambda\tau}}\right)\sqrt{\frac{c\rho\tau}{\lambda}}$. Then

$$T_1 \leqslant \frac{Ev}{4\pi\lambda}\operatorname{erfc}\left(\frac{z\sqrt{c\rho}}{2\sqrt{\lambda\tau}}\right)\exp\left(\frac{\alpha z}{\lambda}\right)\frac{\alpha\sqrt{\tau}}{\sqrt{\lambda c\rho}}. \tag{13}$$

A comparison of (12) and (13) reveals that $T_1/T_0 \ll 1$ (at $\tau < \lambda c\rho/a^2$). This means that a convection effect at τ less than $\lambda c\rho/\alpha^2$ is insignificant. The coefficient α has a pronounced influence on the tissue temperature only at large τ and T when the system attains a stationary regime. Hence we may use formula (12) either without corrections or with an account by source pulses:

$$w(\tau) = \begin{cases} E\delta(r, z), & \tau - \dfrac{[\tau v]}{v} < \tau_p, \\ 0, & \tau - \dfrac{[\tau v]}{v} \geqslant \tau_p, \end{cases}$$

$$T_0 = \frac{E}{4\pi\lambda R}\operatorname{erfc}\frac{R\sqrt{c\rho}}{\sqrt{4\lambda\left(\tau - [\tau v]\left(\frac{1}{v} - \tau_p\right)\right)}}. \tag{12'}$$

For tissues with the absorption coefficient $\kappa < 1000\ \mathrm{m}^{-1}$, it is necessary to consider a spatial source (see Sec. 1). In the general case for a source with β, $l \neq 0$, an analytical solution of the problem may be built only in series. A sufficiently simple formula may be derived if we take $l = 0$ and do not consider heat transfer from the surface. These constraints are valid for the strongly scattering tissue and short process duration.

The set of equations describing such a process may be represented as

$$c\rho\frac{\partial T}{\partial\tau} = \lambda\left[\frac{\partial^2 T}{\partial R^2} + \frac{2}{R}\frac{\partial T}{\partial R}\right] + w, \tag{14}$$

$$T|_{R\to\infty} = 0,$$

$$w = \begin{cases} \dfrac{\varkappa Ev}{2\pi\left[a^2 + \dfrac{\varkappa a^3}{3}\right]}, & R \leqslant a, \\ \dfrac{\varkappa Eva^2\exp[-\varkappa(R-a)]}{2\pi R^2\left[a^2 + \dfrac{\varkappa a^3}{3}\right]}, & R > a, \end{cases} \tag{15}$$

which corresponds to the case $l = 0$ (see Sec. 1).

Using the Laplace transform [6], the solution of the system may be written as

$$\tilde{T}_1 = \frac{w_1}{c\rho s_\tau^2} + c_1 \frac{\operatorname{sh}\left(R\sqrt{\frac{s_\tau c\rho}{\lambda}}\right)}{R}, \quad R \leqslant a,$$

$$\tilde{T}_2 = c_2 \frac{\exp\left(-R\sqrt{\frac{s_\tau c\rho}{\lambda}}\right)}{R} + \frac{1}{R\sqrt{s_\tau^3 c\rho\lambda}} \times$$

$$\times \int_a^R R w_2 \operatorname{sh}\left(\sqrt{\frac{s_\tau c\rho}{\lambda}}(R-t)\right) dt, \quad R > a.$$

The constants c_1 and c_2 are determined from the equations

$$\tilde{T}_1 = \tilde{T}_2|_{R=a}, \quad \frac{\partial \tilde{T}_1}{\partial R} = \frac{\partial \tilde{T}_2}{\partial R}\bigg|_{R=a}.$$

Using Po > 1 at $\tau \to 0$, an expression for the inverse temperature transform may be written in the form

$$T_2 = \frac{\varkappa E\nu a^2 \exp[-\varkappa(R-a)]\tau}{2\pi\left[a^2 + \frac{\varkappa a^3}{3}\right] R^2 c\rho}, \quad R > a. \tag{16}$$

Analysis of the latter form shows that the temperature rapidly decreases as far as the distance from the center of irradiation increases. A major contribution to tissue heating is made by internal sources (absorbed laser radiation); as regards transfer by conduction, its effect is insignificant. Therefore the heating time may be evaluated by using the Fourier and Pomerantsev numbers

$$\text{Po} = \frac{wL^2}{\lambda T}, \quad \text{Fo} = \frac{\lambda\tau}{c\rho L^2}.$$

Then from FoPo $\approx$ 1

$$\tau_h \approx \frac{c\rho T}{w} = \frac{c\rho a^2 T}{\varkappa E\nu}. \tag{17}$$

Figure 2 shows the temperature field distribution calculated by formulas (12), (16) for the following parameters: a = 200 μm, E = 200 mJ, ν = 100 Hz.

3. Comparison with a numerical solution by the method of finite elements. Equation (7) may be solved numerically by the method of finite elements (Fig. 3) with an account of the laser radiation absorption (see Sec. 1). In this case, we solve the two-dimensional axisymmetric nonstationary problem

$$c\rho \frac{\partial T}{\partial \tau} = \lambda\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] + w. \tag{18}$$

Here internal sources w are prescribed in accordance with Sec. 1.

Boundary conditions at z = 0:

$$\lambda \frac{\partial T}{\partial z} - \alpha T = Q, \tag{19}$$

where Q is the density of surface sources in an irradiated circle with radius r. In real conditions, the energy of radiation is described between a surface source and an internal source as

$$Q = \frac{\gamma E \nu}{\pi a^2}, \quad \gamma = \overline{0,\ 1}, \quad w = \varkappa (1 - \gamma) p.$$

A value of γ depends on the heating temperature of the laser scalpel tip (with increasing temperature a portion of heat radiation in the far UV-region increases which then becomes absorbed on the surface) and on a portion of strongly absorbing ($\kappa > 5000\ m^{-1}$) components in the tissue. It may be precisely determined experimentally. With other parameters being equal, an increase of γ entails the rise of the maximum temperature and the temperature field localization in the near-surface region. A decrease of γ causes heating of internal tissue layers. An influence of the length of zone II is less pronounced. An increase of l results in a decrease of the maximum temperature and, simultaneously, in more essential heating of lower layers of epiderma.

The results of the problem considered above may be applied for analysis of a possible use of laser cutting instruments in medicine. Application of analytical models (see Sec. 2) may serve as an estimate for determination of temperature fields with an accuracy of $\pm 30\%$. To determine it more accurately, it is necessary to solve the problem (18), (19) with internal sources (3)-(5) by numerical methods. The results of calculations reveal that a necrosis zone makes up 1-2 mm if the instruments above are used. This imposes some restrictions on their use. A knowledge of thermal conditions of the tissue allows a proper choice of the minimum power of the instrument and the working wavelength of the laser for different tissues.

NOTATION

E, energy of a laser pulse; τ_p, pulse duration; ν, pulse repetition frequency; I, radiation intensity; s, characteristic coordinate along the direction coinciding with the vector Ω; κ, absorption coefficient; σ, scattering coefficient; $\psi(\Omega\Omega')$, scattering indicatrix; r, z, φ, current coordinates; p, radiation power density; λ, thermal conductivity; $c\rho$, volume heat; T, tissue temperature; τ_h, heating time; w, density of internal sources; α, heat transfer coefficient on a surface; Nu = $\alpha L/\lambda$, Nusselt number.

REFERENCES

1. M. N. Otsisik, Combined Heat Transfer [in Russian], Moscow (1976).
2. B. N. Chetverushkin, Mathematical Modeling of the Problems of Emitting Gas Dynamics [in Russian], Moscow (1985).
3. V. S. Aleinikov and V. I. Masychev, Carbon Dioxide Lasers [in Russian], Moscow (1990).
4. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transfer [in Russian], Moscow (1981).
5. G. Korn and T. Korn, Mathematical Handbook for Scientists and Engineers, McGraw Hill, New York, (1977).
6. E. Kamke, Handbook of Ordinary Differential Equations [Russian translation], Moscow (1976).